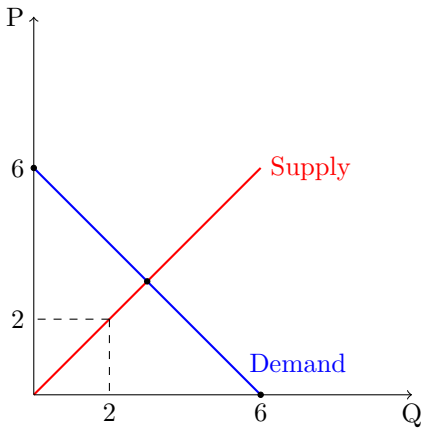
Taxes, minimum price, and revenue

1. The government, fearing that a shortage of titanium could jeopardize national security, establishes a tax of \$2 per ounce on the retail price of this rare metal, to be paid by titanium sellers. The attached graph shows the initial supply and demand curves for titanium. On that same graph, show how the tax will affect the price and equilibrium quantity of titanium in the short term. Clearly indicate all the important points.



2. In the titanium market described in problem 1 (without taxes), assume that a minimum price of \$4 per ounce is established, so only 2 tons of titanium are sold per year (tax-free). Describe a transaction that would improve the welfare of some buyers and sellers without harming others.

3. Suppose that in the titanium market from problem 1, with a tax of \$2 per ounce, the demand for titanium increases due to new medical applications. The new demand curve is $P = 8 - Q$. Find the change in the state's tax revenue caused by the increase in titanium demand.

4. Suppose that in the titanium market from problem 2, without taxes but with a minimum price of \$4 per ounce, the supply decreases due to reduced titanium reserves. The new supply curve is $P = 2 + Q$. How does the excess supply change as a result of the reduced supply? Is the minimum price still relevant (does it raise the price above the equilibrium level)?

Solutions

1. First, we obtain the supply and demand functions. The supply function passes through the origin and has a slope of 45 degrees, so: $P = Q$. The demand function passes through the points $(0, 6)$ and $(6, 0)$, so it also has a slope of 45 degrees, but with an intercept of 6: $P = 6 - Q$. The initial equilibrium is given by:

$$6 - Q = Q$$

$$Q = 3$$

$$P = 3$$

Now, with the tax, a new price is generated for the consumer and the producer:

$$P_d - P_o = 2$$

That is:

$$P_d = 2 + P_o$$

Now, we match supply and demand. The demand is now:

$$P_d = 6 - Q$$

$$2 + P_o = 6 - Q$$

$$P_o = 4 - Q$$

Matching it with the supply:

$$4 - Q = Q$$

$$Q = 2$$

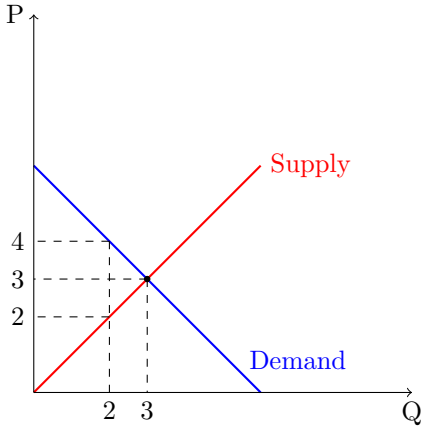
Now, we obtain the two prices:

$$P_o = 2$$

$$P_d = 4$$

Therefore, the state collects $2 \cdot 2 = 4$.

Graphically:



2. If a minimum price of 4 per ounce is set, an excess supply will occur. **A possible transaction would be that, in a parallel and unregulated market, the titanium is sold at a lower price,** since there are sellers willing to sell for less and buyers willing to buy.

   Transaction that improves welfare:

   A buyer who values an ounce of titanium at, for example, \$3.50, and a seller willing to sell it at that price, can make a private transaction at \$3.50 per ounce. This sale benefits both:

   - Buyer: Obtains the titanium at a price they are willing to pay, which is below the official minimum price of \$4.
   - Seller: Sells an additional ounce of titanium at a price above their cost and below the minimum price, increasing their income.

3. Previously, the tax revenue was 4. Now, if demand increases, we need to recalculate the equilibrium with the tax:

$$P_d = 8 - Q$$
$$2 + P_o = 8 - Q$$
$$P_o = 6 - Q$$

   Matching with the supply:

$$6 - Q = Q$$
$$Q = 3$$

   Now, we obtain the two prices:

$$P_o = 3$$

$$P_d = 5$$

   Therefore, the state collects $2 \cdot 3 = 6$. That is, an increase of 2.

4. If the supply is $P = 2 + Q$, the initial equilibrium is:

$$2 + Q = 6 - Q$$

$$Q = 2$$

   This leads to a price of $P = 4$. Therefore, a minimum price of 4 has no effect, as it matches the equilibrium price, and no excess supply is generated. If we consider that a minimum price was set first and then supply decreased, we could say that the excess supply was reduced until it disappeared.